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A Framework for Routing Systems in the Supply Chain Including: The Structure of Vehicle Routing Problems

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A Framework for Routing Systems in the Supply Chain

The Structure of Vehicle Routing Problems

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Abstract

Vehicle routing is a critical element in managing the global supply chain. Reducing costs and ensuring quality is still a challenge in the field of people- and goods- logistics. Much research happens in this field leading to new algorithms for solving the different kinds of vehicle routing problems. Comparing the quality of different approaches established by different teams is still difficult. Therefore a framework as container for algorithms was developed. Researchers will be able to concentrate on developing methods and integrating their solutions in the framework. Further studies would be in the position to be supported and made comparable by this research and development tool, which will be given to universities as a base for further investigations in vehicle routing algorithms. A problem of the newspaper distribution is described and will be presented in the scope of the framework.

A second part of this paper (*The Structure of Vehicle Routing Problems*) describes the core unit of the research project *A Framework for Routing Systems in the Supply Chain*. It covers the specification of the technical requirements of transportation planning as well as the modelling and implementation of a method for optimizing the discrete process which minimizes the total cost of transportation while observing time constraints and vehicle capacities.

Keywords

cluster – cluster-saving – combinatorial programming –heuristic–pickup and delivery– scheduling–heuristic–set covering–set partition–super cluster–vehicle routing

1. A Framework for Routing Systems in the Supply Chain

1.1 Introduction

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Dantzig and Ramser (1959) formulated the Vehicle Routing Problem (VRP). In its basic version, it calls for the construction of a set of routes from a given number of customers. Since this time a huge number of exact and heuristic methods were developed. The original problem was extended with a huge number of restrictions. Toth, Vigo (2002) and Cordeau (2001) distinguish:

Table 1	VRP versions
CVRP	VRP with Capacity Restrictions
VRPTW	CVRP with Time Windows
VRPB	VRP with Backhaul
VRPPD	Pickup and Delivery
VRPPDTW	VRPPD with Time Windows
MDVRP	VRP with Multiple Depots
PVRP	Periodic VRP
PVRPTW	Periodic VRPTW
SVRP	Stochastic VRP

The methods used in practice are developed from academic institutions as well as from route planning suppliers. E.g., the company ILOG (De Backer 2000) concentrates upon optimization methods and offers libraries for modeling and solution of route planning problems based on metaheuristics.

A huge number of practical optimization problems cannot be solved with these standard libraries and still requires an individual problem analysis and academic research. An example for this is the extension of the MDVRPTW with departure time restrictions as they appear in the press distribution. The vehicles cannot leave at the most early possible time because the product to be delivered, i.e. the newspaper, is not available at the beginning. This leads to staggered departure times. We call this planning variation SCVRP (Supply Chain Vehicle Routing Problem).

The purpose of this work is a framework, which illustrates the abovementioned planning variations and makes available the well-known test records in a uniform format. The processing of the planning scenarios occurs within this framework and these data are made

available through an interface. After occurred optimization, the results are evaluated and presented by the framework. Within the scope of this work, new reference records are also provided for the SCVRP.

The Liechtenstein University of Applied Sciences offers free download of the framework. Well-known test records, for example from Solomon, are offered in the form of databases. The framework has access to these databases.

The test of algorithms from other academic institutions is planned as free service. Thus a neutral institution, the Liechtenstein University of Applied Sciences can verify optimization results.

1.2 Architecture

A huge number of VRP packages are available in the market. They consists of three layers

- 1. Graphical Information System (GIS)
- 2. Administration
- 3. Operations Research Methods

The framework follows this architecture.

The GIS layer is responsible for distance calculation, geocoding (the process of assigning a customer to the traffic-network by his address) and graphical representation of the calculated results. The street network is delivered by the company's Teleatlas or Navtech. As a new actor in this market Microsoft is recently active. The GIS program MapPoint is offered which has a complete automation interface and can be used therefore as a component.

The Framework is able to use Euclidean calculations, however, can use alternatively also the MapPoint distance calculation based on the street network. This should close the gap between theory and practice as the theoretical reference cases use primarily Euclidean distances. In practice a distance calculation is used based on shortest path methods on the underlying street network.

The administration layer manages the business data and visualises the results. Furthermore the administration layer should yield the possibility to manipulate the calculated results via drag and drop to react on short-term problems of the route plan. E.g. auxiliary functions offer the possibility to integrate unplanned deliveries in the route plan.

The optimization data as well as the results are stored in a Microsoft or Oracle SQL-database.

The algorithms were integrated by means of a Web Service. The optimization data such as customers, deliveries, depots and vehicles are transferred to the OR-method. The use of the Web Service technology requires an application server like Tomcat or Internet Information Server (IIS). The communication via SOAP and HTTP allows the integration of algorithms that do not run in the intranet, i.e. from external academic institutions. The influence of an application server on the velocity of the OR-method will be investigated. Another advantage that we see is the independence of a concrete computer language. The web interface must be supported what is a standard feature of current computer languages (C#, Java, Delphi).





1.3 Test data

A huge number of test records for the base problems are available. They have different formats according to their origin. The described test records were processed in a uniform database structure. Only Euclidean data were taken into consideration.

1.3.1 CVRP Toth-Vigo

Toth and Vigo (2002) have processed test data for their work in the area of CVRP. According to the convention of Vigo (2000) the records are named in the form tnnnvkkp.

t	E euclidean
	A asymmetric
	D symmetric
nnn	Number of nodes in the graph
V	Separation character
kk	Number of vehicles
р	a Hays and Eilon, Watson-Gandy and Christofides
	c Christofides, Mingozzi and Toth
	d Dantzig and Ramser and Eilon, Watson-Gandy and Christofides
	e Christofides and Eilon
	f Fisher
	g Gaskell and Eilon, Watson-Gandy, and Christofides
	h Hadjiconstantinou, Christofides, and Mingozzi
	k Golden et.al
	m Chistofides, Mingozzi, and Toth
	n Noon, Mittenthal, and Pillai
	v Fischetti, Toth, and Vigo
	w Clarke and Wright and Eilon, Watson-Gandy and Christofides

Table 2VRP-data according to Vigo (2000)

1.3.2 VRPTW Solomon

The records of Solomon (1987) are divided into six different groups and contain in each case 100 customers. These test records were extended by Homberger and Gehring (1999) to 200, 400, 600, 800 and 1000 customers.

Table 3	VRPTW-classes according to Solomon
R1	Customers are steadily distributed with short planning horizon
C1	Customers are geographically clustered with short planning horizon
RC1	Customers have R1 type or C1 type
R2	Customers are steadily distributed with long planning horizon
C2	Customers are geographically clustered with long planning horizon
RC2	Customers have R2 type or C2 type

1.3.3 MDVRP Gillett, Johnson; Chao; Chao Golden und Wasil

For MDVRP 23, classical test problems exist from Gillett and Johnson (1967), Chao (1993) and Chao, Golden and Wasil (1993). The problems contain between 50 and 360 customers and two to nine depots.

1.3.4 MDVRP Cordeau, Gendreau und Laporte

Other ten test problems MDVRP come from Cordeau, Gendreau and Laporte (1997). They contain between four and six depots, between 48 and 288 customers and between two and twelve vehicles per depot.

1.3.5 MDVRPTW Cordeau, Laporte, Mercier

The test problems 1.2.3 were extended by Cordeau, Laporte and Mercier (2001) with time windows. Here two variations were provided with identical geographical data but different time windows.

1.3.6 VRPPD Li & Lim

Li and Lim (2002) provide test records with 100, 200,400, 600, 800 and 1000 customers. The dataset is generated out of the Solomon (1987) and Homberger, Gehring (1999) VRPTW cases and has six problem classes C1, R1, RC1, C2, R2, RC2.

1.4 SCVRP Vehicle Routing Problem in the Supply Chain

With the help of the framework a solution for a German forwarder will be developed. It concerns a problem of MDVRPTW type with an additional restriction which arises from the production of the press articles.

1.4.1 Influence of Production

The production of the press products is carried out at a location near Hannover and starts against 20:00. The printing lines deliver constantly between 15000 and 18000 newspapers per hour. They are loaded onto vehicles, driven to depots, and are further distributed. This leads to the necessity to let drive off the vehicles at the printing location as well as from the depots temporally staggered.

1.4.2 Time Windows

The observance of the given narrow time window is a critical restriction in this problem class. If e.g., newspapers are delivered too late in the railway station bookshop they cannot be sold. The time windows are put very narrowly and determine essentially the structure of the result.

1.4.3 Vehicles

At the printing location as well as in the depots different vehicle classes are available. Therefore, e.g., a smaller vehicle at the printing location can drive off earlier, because it is fully loaded at an earlier time. If smaller vehicles are used, the costs will increase, on the other hand the product can be delivered earlier and therefore the temporal restrictions can better be fulfilled.

1.4.4 Literature Review

The SCVRP-Problem was not paid much attention to. The research from Holt, Watts (1988) and Hurter, Van Buer (1996) describe single level distribution (i.e. transport from the printing location directly to the destination locations) and investigate staggered departure times as an effect of the production times. Jacobsen und Madsen (1980) optimize a SCVRP problem. Additionally they determine the most favorable depot points (instead of fixed depots). Modern metaheuristics like Tabu-Search (Glover 1993) are not used.

Figure 2 The SCVRP-problem



2. The Structure of Vehicle Routing Problems

2.1 General Structure of the VRPPD

In its basic form the problem consists of a fleet of vehicles and a set of customer requests. Each request specifies an origin location, a destination location and some particular side constraints. The vehicles must travel through the locations so that each origin is visited before the corresponding destination. In practice the 'Pickup and Delivery Problem' (VRPPD) has a number of variations, but its basic structure is common to all variants of the problem (Savelsbergh 1995).



Figure 3 Structure of a Vehicle Routing Problem

The VRPPD is called a 'Vehicle Routing Problem' (VRP) if either all pickup-nodes or all delivery-nodes are located at a common vehicle location (central hub, depository-node); i.e. either goods or persons are distributed by the depository (0^+) on the certain final destinations (Fig. 3), or they are loaded at their respective origin place and transported to the depository (0^-) .

Figure 4 Structure of a Standard Pickup and Delivery Problem



At the 'Standard Pickup and Delivery Problem' (SVRPPD) every route also starts and ends at the vehicle location (0) but the pickup- and delivery nodes can be put arbitrarily (Fig. 4). Another generalisation of this problem structure leads to the 'General Pickup and Delivery Problem' (GVRPPD) where the vehicles are available in arbitrary places (Fig. 5). The central node (0) serves now as a virtual location for forwarding agency vehicles which can be used at additional vehicle need.

GVRPPD GVRPPD solution 8+ 8+ 8 8-0 1-3+ 3+ 1-3-5-5-1 +44 1 Δ 7+ Ω 0 0 2 +▶₀7-2 +0 0 0 6-C 6-5+ 5+ 2-2-C 6+ 6+

Figure 5 Structure of a General Pickup and Delivery Problem

2.2 Definition of the VRPPD

Let D = (V, A) be a digraph, $V = \{0\} \cup N \cup M$ a set of locations and $A \subseteq V \times V$ its set of links. The central node 0 denotes a virtual depot as well as a location for subcontracting (forwarding agency) vehicles which can be used at additional vehicle need. $N = N^+ \cup N^-$ covers the origins $i^+ \in N^+$ and destinations $i^- \in N^-$ of all transportation requests. $M = M^+ \cup M^-$ represents the set of all start locations $k^+ \in M^+$ and end locations $k^- \in M^-$ of the vehicles of a fleet F. Each link (a, b) $\in A$ is weighted by two functions c: $A \rightarrow \angle$, t: $A \rightarrow \angle$ which indicates the cost c_{ab} and the time t_{ab} of the travel from point a to point b.¹

The customer requests are characterised by the set $N^{\times} = \{ (i^+, i^-) | i^+ \in N^+, i^- \in N^- \}$. Each pair $(i^+, i^-) \in N^{\times}$ is assigned its requested load $q_i \in \angle$ polarised in the positive quantum q_i for the pickup location i^+ and the negative quantum $-q_i$ for its joined delivery location i^- . Each vehicle $k \in F$ is bounded by a capacity $Q_k \in \angle$.²

Definition 1 A pickup and delivery route for vehicle $k \in F$ is a directed trail $D_k = (V_k, A_k) \subseteq D$ starting in k^+ and stopping in k^- such that for all $i \in N$:

- *1.* if $i + \in V_k$ then $i \in V_k$,³
- 2. *if* $i + \in V_k$ then location i + is visited before location $i , {}^4$
- 3. the vehicle load never exceeds Q_k .

² V is enumerated by the interval [0...2n+2m], $|V|=1+|N|+|M|=1+|N^+|+|N^-|+|M^+|+|M^-|=2n+2m+1$, i.e. the problem deals with $|N^{\times}| = |N^+| = |N^-| = n$ requests and $|F| = |M^+| = |M^-| = m$ vehicles.

³ i.e. $(\{i^+\} \cup \{i^-\}) \cap V_k = \emptyset$ or $(\{i^+\} \cup \{i^-\}) \cap V_k = (\{i^+\} \cup \{i^-\})$.

¹ The time function t_{ab} is represented by the time-distance matrix T = (t[a,b]) which arises from a shortest path calculation of the given road network. Shortest paths can be solved very efficiently. For static purposes T is computed completely in advance by the Floyd-Warshall algorithm (runs in $O(|V|^3)$ time, requires $O(|V|^2)$ space). For meeting dynamic aspects of the vehicle scheduling problem, T must be adapted partially at certain times during the optimisation process by Dijkstra's algorithm (which updates a single shortest path in $O(|V|^2)$ time). The cost function c_{ab} can be composed of weighted sub-functions (e.g. fixed-, time- or distance-costs) according to the business management objective.

 $^{^{4}}$ Each location in V_k is visited by vehicle k exactly once is a feature of a 'trail' according to its definition in the Graph Theory.

A generalised form arises out of definition 1 if in part 3 (the feasibility of the vehicle load) a function is used which observes all constraints, including the capacities of the vehicle and the requests.

Definition 2 A pickup and delivery plan P is a set of pickup and delivery routes D_k such that:

- 1. $P = \{ D_k | k \in F \}$
- 2. $\{A_k | k \in F\}$ is partition of $N^{\times,5}$

Definition 3 The Pickup and Delivery Problem is finding a plan S with the lowest prize c_S corresponding to a certain objective function $c: \Omega(D) \rightarrow \angle$ such that $c_S = \min\{c_P | P \in \Omega(D)\}$ where P is a plan of the set $\Omega(D)$ of all pickup and delivery plans.

There are many variants of the pickup and delivery problem structure. According to the definitions 1–3 the simplest one is called 'Standard Pickup and Delivery Problem' (SVRPPD) if all routes are cycles starting and ending at the central depot 0. The most important type is called 'Multiple Depot Pickup and Delivery Problem' (MDVRPPD) if the routes start and end at arbitrarily situated depots. Mainly from a theoretical point of view the MDVRPPD can be generalised by the 'General Pickup and Delivery Problem' (GVRPPD) in the way that a request and its load are subdivided (Savelsbergh 1995).⁶ A Pickup and Delivery Problem is called 'Dial-A-Ride Problem' (DARP) if persons instead of goods are transported (Savelsbergh 1995). A DARP has its specific requests, such as short planning horizons and fewer orders with very short time intervals and smaller capacities (Weinmann 1998).⁷

⁵ The edge-oriented formulation considers a customer request as a complete object. The node-oriented model is focussed on the locations as the graphical components of a request: { $V_k | k \in F$ } is partition of N.

⁶ Often a complex request of a GPDP can be substituted by several independent single requests, even in cases of subdivided requests has to be served by a single vehicle.

⁷ Similar problem variants occur also at the transportation of animals.

2.3 Common Set Partitioning of the VRPPD

Numerous optimization tasks from different areas of technology and economics are based on a common problem structure. Its solution contains the partitioning of a set of objects under economic points of view. The goal of the investigation is to gain a high capacity utilization rate of the corresponding system. In many cases the quality of a partition depends on the measure in which a favorable sequence (permutation) of the elements within the cluster can be reached. The described problem is an instance of the ordered set partition problem, where the order of the sets (assignment problem)⁸ as well as the order of the objects within the sets (sequencing problem)⁹ matters (Weinmann 2000).

Let N be a finite set of objects, $R \in 2^N$ a subset of N and c: $2^N \rightarrow \angle$, f: $2^N \rightarrow \{0,1\}$ functions which indicates the costs c(R) and the feasibility f(R)=1 of a cluster R. Find a cost minimal feasible super-cluster S of N:¹⁰

$$\min\sum_{R\in S} c(R) \tag{2.1}$$

subject to

$$\sum_{R \in S} f(R) = |S| \tag{2.2}$$

$$(\forall X \in S)(\forall Y \in S - \{X\})[X \cap Y = \emptyset]$$
(2.3)

$$\emptyset \notin S$$
 (2.4)

$$\cup S = N \tag{2.5}$$

Set partitioning structures arises in scheduling fleets of vehicles or aircraft and in many other problems, as setting up networks without causing any capacity transgressions, fault testing of electronic circuits or locating fire stations in urban areas. The Set Covering Problem has the same form except the clusters need not to be disjoint (2.3). This would mean for the VRP that orders don't have to be fulfilled absolutely in just one route. The linear programming formulation expresses the problem of selecting a set of columns (sets) that together cover all rows (elements) at minimum cost. Although in many instances the LP relaxation is known to give solutions close to optimality (if application-specific restrictions can delimit the number of the feasible solutions), the combinatorial latitude generally remains enormously large so that heuristic procedures to the solution of these hard problems should be applied (Rosen

⁸ The Resources Assignment Problem corresponds to the Bin Packing Problem if there are identical capacities.

⁹ Sequencing (of any requests) problems are mostly covered by the Travelling Salesman Problem.

¹⁰ A super-cluster S is a partition of A, i.e. a pair wise disjoint family of nonempty subsets whose union is A.

2000). Furthermore the Bin Packing Problem and all types of the GVRPPD, even the simplest variant, the 'Travelling Salesman Problem' (TSP), are NP-hard (Lenstra 1981). The current state of knowledge in the area of the Algorithms and Complexity Theory doesn't give any certainty about the P-NP Problem yet. A heuristic attempt at the solution of the GVRPPD therefore seems appropriate in addition.

2.4 Heuristic approaches to the VRPPD

Some heuristic methods use variants of strategies which are successful applied to the simplest type of the discussed problem structure, the VRP.

Figure 6 Initial status and first step of a savings heuristic solving a VRP



The savings method in its basic form, using concatenation steps with a grade of complexity $O(|N|^2)$, gain useful results just for the case of a VRP (Fig. 6) or a 'Full Load VRPPD' where a customer request has the same quantity as the capacity of the assigned vehicle (Weinmann 1998). Dethloff (1994) therefore proposed insertion functions in combination with the savings method with an effort $O(|N|^3)$. Weinmann (1998) introduced a generalised variant with a complexity $O(|N|^4)$ applying different insert operations and weighted parameters which prevent optimisation processes in principle from ending in sub-optimal solutions by a less myopic clustering.¹¹

¹¹ A savings method is primarily a *greedy heuristic* which produces an optimal set over a *matroid* (Whitney, 1935)

2.5 Transformation of the MDVRPPD to the SVRPPD

Formulations and solutions of the Standard Pickup and Delivery Problem are often discussed and well described in many research reports, scientific volumes and monographs (Toth and Vigo 2002). Since algorithms for a SVRPPD can be designed with less effort as algorithms for a MDVRPPD, a transformation of the MDVRPPD to the SVRPPD is particularly useful. Reducing a MDVRPPD to a SVRPPD means replacing the start nodes $k^+ \in M^+$ of all vehicles by the central location 0 such that each vehicle $k \in F$ is available at its origin location k^+ at its origin start time T^k_{α} . Real routes from each k^+ to k^- are modelled by virtual trips starting and ending at the centre 0. To make sure that vehicle k starts at its origin k^+ not later than at the time of T^k_{α} and stops at its destination k^- not later than at the time of T^k_{β} , the virtual trip mustn't start at 0 later than at the time of T^k_{α} - t_{0k+} and mustn't end at 0 later than at the time of $T^k_{\beta} + t_{k-0}$.

Let $[t_{\alpha}^{i+}, t_{\beta}^{i+}]$ and $[t_{\alpha}^{i-}, t_{\beta}^{i-}]$ be time windows of request $i = (i^+, i^-)$ where t_{α}^{i+} specifies the earliest, t_{β}^{i+} the latest possible pickup time and t_{α}^{i-} the earliest, t_{β}^{i-} the latest possible delivery time. The respective maximum pickup and delivery time spaces of request i are $\delta^{i+} = t_{\beta}^{i+} - t_{\alpha}^{i+}$ and $\delta^{i-} = t_{\beta}^{i-} - t_{\alpha}^{i-}$. Let $k \in F$ be a vehicle at its origin location $k^+ \in M^+$ which doesn't start before the time of T_{α}^k and doesn't end at its destination $k^- \in M^-$ after the time of T_{β}^k . A route of the vehicle k therefore cannot last longer than $\Delta^k = T_{\beta}^k - T_{\alpha}^k$.



Figure 7 Transformation of a route between two depots k^+ , k^- to a route of a central depot

Let 0 denote the (virtual) central depot and $\mathbf{u} = (\mathbf{u}^+, \mathbf{u}^-)$, $\mathbf{v} = (\mathbf{v}^+, \mathbf{v}^-)$, $\mathbf{w} = (\mathbf{w}^+, \mathbf{w}^-)$ be (virtual) requests of $\mathbf{M}^{\times} = \{ (\mathbf{i}^+, \mathbf{i}^-) \mid \mathbf{i}^+ \in \{0\} \cup \mathbf{M}^+$, $\mathbf{i}^- \in \{0\} \cup \mathbf{M}^- \}$. An equivalent SPDP formulation can be constructed as follows: $\mathbf{u} = (0, \mathbf{k}^+)$, $\mathbf{v} = (\mathbf{k}^+, \mathbf{k}^-)$, $\mathbf{w} = (\mathbf{k}^-, 0)$ are (vehicle-) requests without loads but with strict time constraints which ensure leaving and arriving at the locations 0, \mathbf{k}^+ , \mathbf{k}^- , 0 on the dot: $\delta^{\mathbf{u}^+} = \delta^{\mathbf{v}^-} = \delta^{\mathbf{v}^-} = \delta^{\mathbf{w}^-} = \delta^{\mathbf{w}^-} = 0$, $t_{\alpha}^{u^+} = T_{\alpha}^k - t_{0\mathbf{k}^+}$, $t_{\alpha}^{v^+} = T_{\alpha}^k$, $t_{\alpha}^{w^+} = T_{\beta}^k$.

The additional requests u and w are positioned as the first arc and as the last arc in a SVRPPD route of vehicle k: $(0, k^+, i^+, ..., i^-, k^-, 0)$. Just v has the flexibility of the vehicle time limits to fulfil customer requests which are located in the area of origin k^+ and destination k^- (Fig. 7).

If a vehicle j starts and ends at the same point $(j^+ = j^-)$, the transformation requires merely two requests $u = (0, j^+)$, $w = (j^-, 0)$ with its time windows $t_{\alpha}^{u+} = t_{\beta}^{u+} = T_{\alpha}^{j} - t_{0j+}$, $t_{\alpha}^{u-} = t_{\beta}^{u-} = T_{\alpha}^{j}$, $t_{\alpha}^{w+} = t_{\beta}^{w+} = T_{\beta}^{j}$, $t_{\alpha}^{w-} = t_{\beta}^{w-} = T_{\beta}^{j} + t_{j-0}$ (Fig. 8). No additional requests are required if the simplest (SVRPPD) case appears where a vehicle starts and ends at the centre $(j^+ = j^- = 0)$.

Figure 8 Transformation of a MDVRPPD to a SVRPPD



The node 0 can be an arbitrary geocoded position in the traffic-network such as the initial position of a vehicle or it also can be defined as the location of a forwarding agent. This pragmatic approach for a MDVRPPD is primarily usefully in the case, that an application has merely an algorithm for the VRPPD. So the solution of the extended problem can be obtained by an expansion of the input data combined with the selection of the result data (by filtering the virtual transformation orders out). The available algorithm doesn't have to be replaced by a more expensive one.

3. References

- De Backer B., Furnon V., Shaw P., Kilby P, Prosser P. (2000) Solving Vehicle Routing Problems Using Constraint Programming and Metaheuristics. *Journal of Heuristics*, Vol 6, pp. 501-523.
- Chao I-M. (1993) Algorithms and Solutions to Multi-Level Vehicle Routing Problems. Dissertation, Applied Mathematics Program, University of Maryland, College Park, MD.
- Chao I-M., Golden B.L., Wasil E.a. (1993) A new Heuristic for the Multi-Level Vehicle Routing Problems.that improves upon best-known Solutions. *American Journal of Mathematical and Management Science*, Vol. 13, pp. 371-406.
- Cordeau J.F., Laporte G., Mercier A. (1997) A Tabu Search Heuristic for Periodic and Multi-Depot Vehicle Routing Problems. *Networks*, Vol.30, pp. 105-119.
- Cordeau J.F., Laporte G., Mercier A. (2001) A Unified Tabu Search Heuristic for Vehicle Routing Problems with Time Windows. *Journal of the Operational Research Society*, Vol 52, S. 928-936.
- Cordeau J.F., Laporte G. (2001) A Tabu Search Heuristic for Site Dependent Vehicle Routing Problem with Time-Windows. *INFOR*, Vol. 39 (2001), pp. 292-298.
- Dantzig G.B, Ramser.J.H. (1959) The truck dispatching problem. *Management Science*, Vol. 6, pp. 80-91.
- Dethloff, J. (1994) Verallgemeinerte Tourenplanungsprobleme. Vandenhoeck&Ruprecht.
- Gillett B.E, Johnson J.G. (1967) Multi-Terminal Vehicle Dispatch Algorithm. *Omega*, Vol. 4, pp. 771-718.
- Glover F., Laguna M. (1993) Tabu Search. Modern Heuristic Techniques for Combinatorial Problems, Halstedt Press, New York pp. 50-150.
- Holt J.N, Watts A.M. (1988) Vehicle routing and scheduling in the newspaper industry. In Golden and Assad, editors, *Vehicle Routing: Methods and Studies*, North-Holland, Amsterdam, pp. 347-358.
- Homberger J, Gehring H. (1999) Two Evaluationary Metaheuristics for the Vehicle Routing Problem, FernUniversität Hagen.
- Hurter A.P., Van Buer M.G. (1996) The newspaper production/distribution problem. *Journal* of *Business Logistics*, Vol. 17 pp. 85-106.
- Jacobsen S.K., Madsen O.B.G. (1980) A comparative study of heuristics for a tow level routing-location problem. *European Journal of Operational Research*, Vol. 5 pp. 378-387.
- Lenstra, J.; Rinnooy Kan, A. (1981) Complexity of Vehicle Routing and Scheduling Problems. *Networks 11*, S. 221-227.
- Li H., Lim A. and Lim H. (2002) Solving the pickup and/or delivery problem, 6th Pacific Asia Conference on Information Systems (PACIS2002), Tokyo, Japan.
- Rosen, K.H. (2000) Handbook of Discrete and Combinatorial Mathematics. Boca Raton: CRC Press LLC, pp. 1003-1004.

- Savelsbergh M. W. P., Sol, M. (1995) The General Pickup and Delivery Problem. *Transportation Science 29(1)*, pp. 17-29.
- Solomon M.M (1987) Algorithms for the vehicle routing and scheduling problems with time window constraints. *Networks*, 35:254-265.
- Toth P., Vigo D. (2002) The vehicle routing problem. *SIAM Monographs on Discrete Mathematics and Applications*, Philadelphia.
- Vigo D. (2000) VRPLIB: A vehicle routing problem instances library. *Technical Report* OR/00/3, Università di Bologna, Italy.
- Weinmann, S. (1998) Dial-A-Ride: Ein Verfahren für die computergestützte Tourenplanung der Personenlogistik. Universität Bremen: Institut für Informatik und Verkehr, pp. 82.
- Weinmann, S. (2000) Information als Erfolgsfaktor: Cluster-saving Eine konstruktive Methode des Operations Research. Stuttgart, Leipzig, Wiesbaden: B. G. Teubner, pp. 59-76.