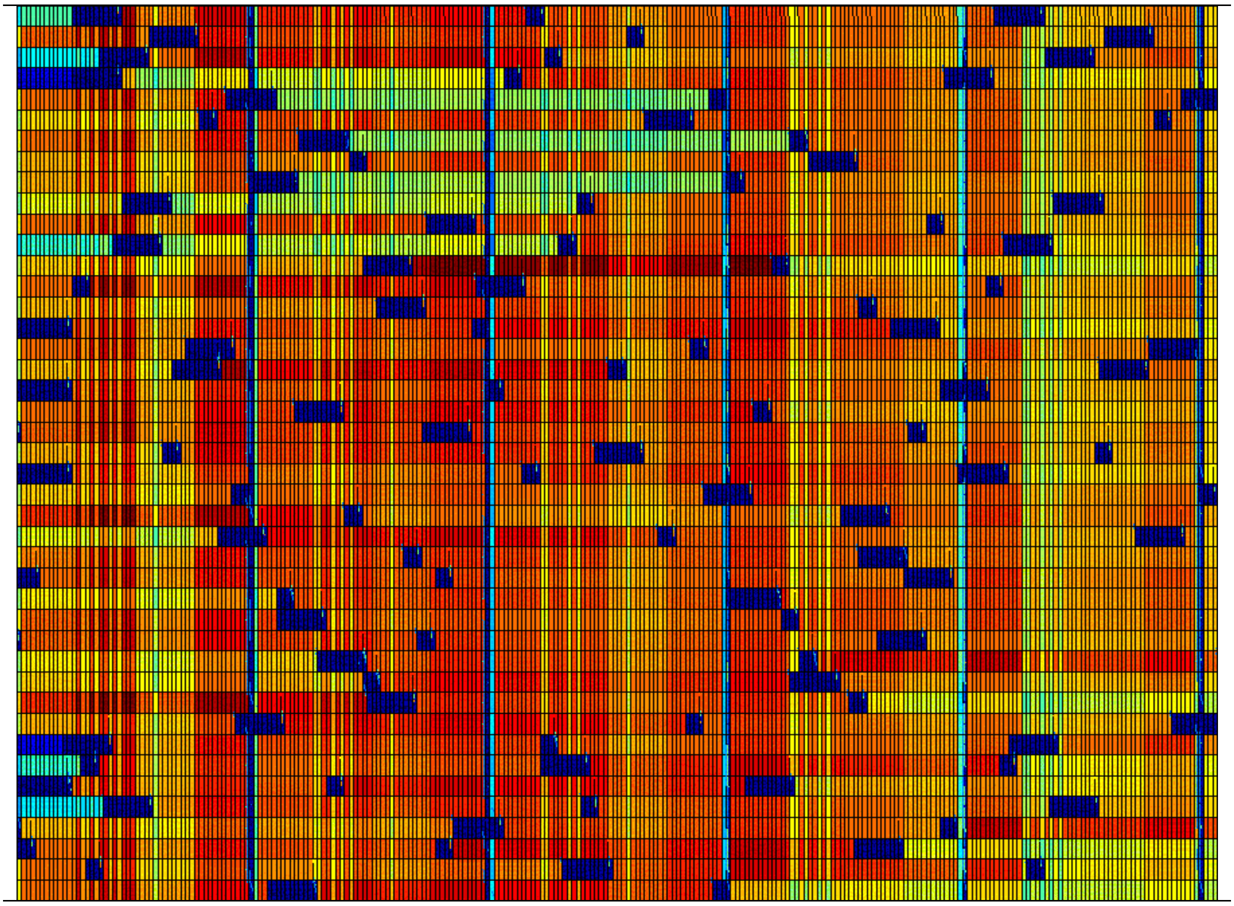

A Heuristic Method for Aircraft Maintenance Scheduling under Various Constraints

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Abstract

Aircraft maintenance is a very important aspect of aircraft fleet management since it usually accounts for a substantial part of the overall operational costs and sets constraints on the planning of flight operations. Maintenance scheduling underlies typically a large number of constraints. Among them are capacities of maintenance facilities, capacities, and skills of maintenance staff, fleet-specific maintenance rules as well as inter-maintenance flying hours and quarterly flying hour demands.

In this paper, we present a novel heuristic method for preventive aircraft maintenance scheduling which has been developed in a joint project of the Institute of Data Analysis and Process Design (IDP) and the Swiss Air Force (SAF).

For two fleets we show some results and findings. The algorithms have shown to work very reliable, fast, and with good optimisation results even with strong constraints, e.g. with various manual settings. One of the major benefits is a significant increase in speed to compute a new maintenance/flight plan (now within 5 to 15 minutes; before: 1.5 to 2 days). This allows for a fast reaction on events like thunderstorms (Bernese Oberland, August 2005), tsunamis (Sumatra mission, January/February 2005) etc. Moreover, investigations of 'What-If-Analyses' to compare different maintenance strategies can now be carried out efficiently.

Keywords

Maintenance scheduling – Heuristic optimisation – Fleet management – Aircraft maintenance

1. Introduction

Aircraft maintenance is a very important aspect of aircraft[1] fleet management since it usually accounts for a substantial part of the overall operational costs and sets constraints on the planning of flight operations. Maintenance scheduling underlies typically a large number of constraints. Among them are capacities of maintenance facilities, capacities, and skills of maintenance staff, fleet-specific maintenance rules as well as inter-maintenance flying hours and quarterly flying hour demands. Moreover, the costs of a specific maintenance action (MA) are not constant in general, but may depend both on time and on capacity utilisation, since additional work force or maintenance facilities may lead to extra costs.

Due to the large number of constraints and the complexity of costs, maintenance scheduling problems are normally time-consuming to solve, especially for large fleets. Additionally, maintenance plans, even for fleets servicing scheduled flights, are hard to follow due to changes in flight plans or corrective maintenance actions on aircraft components. This holds even more for fleets servicing unscheduled flights (ambulance, air force flights, leisure etc.). Fleets allocated for such flights are typically facing highly variable and unpredictable demand. Therefore, maintenance plans have to be updated frequently. From this, it follows that the generation of a maintenance plan must be fast and efficient.

The ten different fleets considered in this project consist of 10 to 50 aircrafts. Depending on the fleet, the required maintenance capacities can be substantially reduced by performing some calendar-based maintenance actions (CBMAs) and usage-based maintenance actions (UBMAs) at the same time, i.e., merging specific UBMAs and CBMAs.

Plans are usually set up for a period of five years. However, deviations from flight plans are common due to corrective actions, urgent missions, poor weather conditions etc. Thus, deviations from the nominal maintenance plan become soon too large and therefore the maintenance plan has to be updated frequently. This is tedious and time-consuming, especially with a semi-automated procedure as it was used before the presented method was introduced.

In 2002, the Swiss Air Force (SAF) evaluated commercially available tools for maintenance scheduling that cover their specific needs. However, none of them was capable of meeting the various requirements given by the operation of the SAF fleets.

[1] For the sake of simplicity, we will use aircraft for both aircraft *and* helicopter throughout this document.

The major aim of this project was to develop a methodology to schedule maintenance actions so that (i) the overall number of MAs is minimised and (ii) the capacity requirements and flying hours are distributed evenly in time. Additionally, a customer-specific software tool was developed in which the algorithms were integrated. The various intentions of the project are summarised in Table 1.

Table 1 Project aims

- Maintenance/flight plans that include:
  - Maintenance actions per aircraft, and
  - Flying hours per aircraft per week required to cope with the plan

  These plans must meet the various operational constraints according to Table 2 (see section 2 and the explanations therein).
- The resulting maintenance capacity requirements per fleet should have a minimum number of exceedings of the limits and a small variation over time
- The flying hours per aircraft shall also have low variation throughout the planning horizon
- Substantial reduction of the time to generate a new maintenance plan
- Maximising the number of merged maintenance actions and therefore minimising the maintenance time (this holds of course only for fleets with both CBMAs and UBMAs)
- Developing a software tool, with:
  - An interface to the existing business software environment, i.e., a connection to the ERP[2] software system (SAP); besides the import of ERP data, this includes also semi-automated data pre-processing
  - Interaction capabilities via GUI (e.g., allowing manual settings of maintenance actions, decommissionings etc.)
- Implementation of an algorithm such that:
  - All fleets can be handled the same way
  - Specific knowledge and experience of operators is incorporated

  Both aspects allow for a user-independent operation, i.e., every fleet can be managed by any operator.

In the subsequent sections we present a novel heuristic method for preventive aircraft maintenance scheduling, which has been developed in a joint project of the Institute of Data Analysis and Process Design (IDP) and the SAF.

The document is structured as follows: In section 2, we discuss the various constraints considered and in section 3, we describe in detail the methodology developed. Although there are some important fleet-specific requirements, the general procedure is very similar for all

[2] ERP: Enterprise Resource Planning

fleets. In section 4, we present the results achieved for some of the fleets. Section 5 concludes with a summary, lists the customer benefits, and gives a short outlook on further research on this topic.

2. Constraints

As already mentioned, maintenance scheduling is subjected to a large number of constraints. In Table 2 below, we list the constraints considered in this project. We identified four groups, to which we can assign the following constraints:

- Constraints that are defined by the MAs themselves (#1 to #4 in Table2)
- Constraints that are defined through manual settings by the operator (i) global settings (#6, #7 and #10), and (ii) interactive settings (#5 and #9)
- ERP data up to the time of scheduling (#8); ERP data comprehend information on (i) remaining flying hours until the next MA, type of the running UBMA and/or CBMA (if any) together with its finish date, (iii) type of the next UBMA and (iv) type and due date of the next CBMA (if any).
- General specifications (#11)

To get meaningful maintenance plans, the following constraints must be relaxed to a certain extent:

- Quarterly flying hour requirements (#6 in Table 2): The quarterly flying hour budgets are important for planning purposes. Due to frequent changes in flight plans or corrective maintenance actions on aircraft components, it is meaningful to allow deviations within a certain range. This is important since it allows to find a reasonable solution that meets the strict constraints.
- Maximum flying hours per aircraft per week (#7): This constraint has (i) a large influence on the deviations from the quarterly flying hours, and (ii) a minor influence on the distribution of the flying hours within the planning horizon.
- Maximum maintenance capacities (#10): Specific limits on maintenance capacities exist for each location. External capacities can be purchased to overcome temporal shortages. As shown in Table 1, the required maintenance capacities as well as the flying hours per aircraft per week are to have a low variation. Thus, short exceedings of the available capacities are accepted since it allows the algorithm to find a feasible solution.
- Shifting tolerances (#2, for UBMAs only): For all fleets, the tolerance ranges for shifting UBMAs forward and backward, respectively can be chosen such that a maximum number of mergings is possible.

Table 2 Constraints considered in this project.

#	Constraint	Inst.[3]	Type[4]	Description
1	Inter-maintenance flying hours	M	S	The number of flying hours between two consecutive maintenance actions is important regarding safety aspects.
2	Shifting tolerances of maintenance actions	M	S	Each MA can be shifted within a defined tolerance range.
3	Sequence and duration of maintenance actions	M/A	S	The sequence of the maintenance actions is defined by the aircraft manufacturer and the SAF. The duration of the MAs depends on the working process and on the available work force at the local air bases.
4	Rules for maintenance mergings	M/A	S	For fleets with both UBMAs and CBMAs, some defined combinations of MAs can be merged!
5	Fixed special services	M/A	S	Tasks like aircraft upgrades (e.g., upgrade of electronic components) are defined usually by the manufacturer and the SAF, whereas decommissionings are planned by the SAF only.
6	Quarterly flying hour requirements	A	R	To meet the long-term requirements of the SAF (flight trainings, military services) as well as of external institutions (for transportation or rescue missions etc.), the compliance with nominal flying hours per fleet per quarter, is of great importance.
7	Max. flying hours	A	R	Maximum flying hours per aircraft per week
8	ERP data	A	S	The daily changing ERP data contain aircraft specific information like remaining flying hours, upcoming MAs with their due dates etc.
9	Fixed flying hours and/or MAs	A	S	To have maximum flexibility, flying hours, and/or MAs can manually be fixed to certain periods by the operator (within their allowed tolerance ranges).
10	Available maintenance capacities	A	R	To conduct the MAs, only a limited number of facilities and personnel ($c_t^{\lim}$) are available.
11	Restrictions due to public holidays	-	S	For the allocation of flying hours, the number of public holidays per week is considered.

[3] Decisive institution for specifying constraints: M → Aircraft manufacturer, A → Swiss Air Force

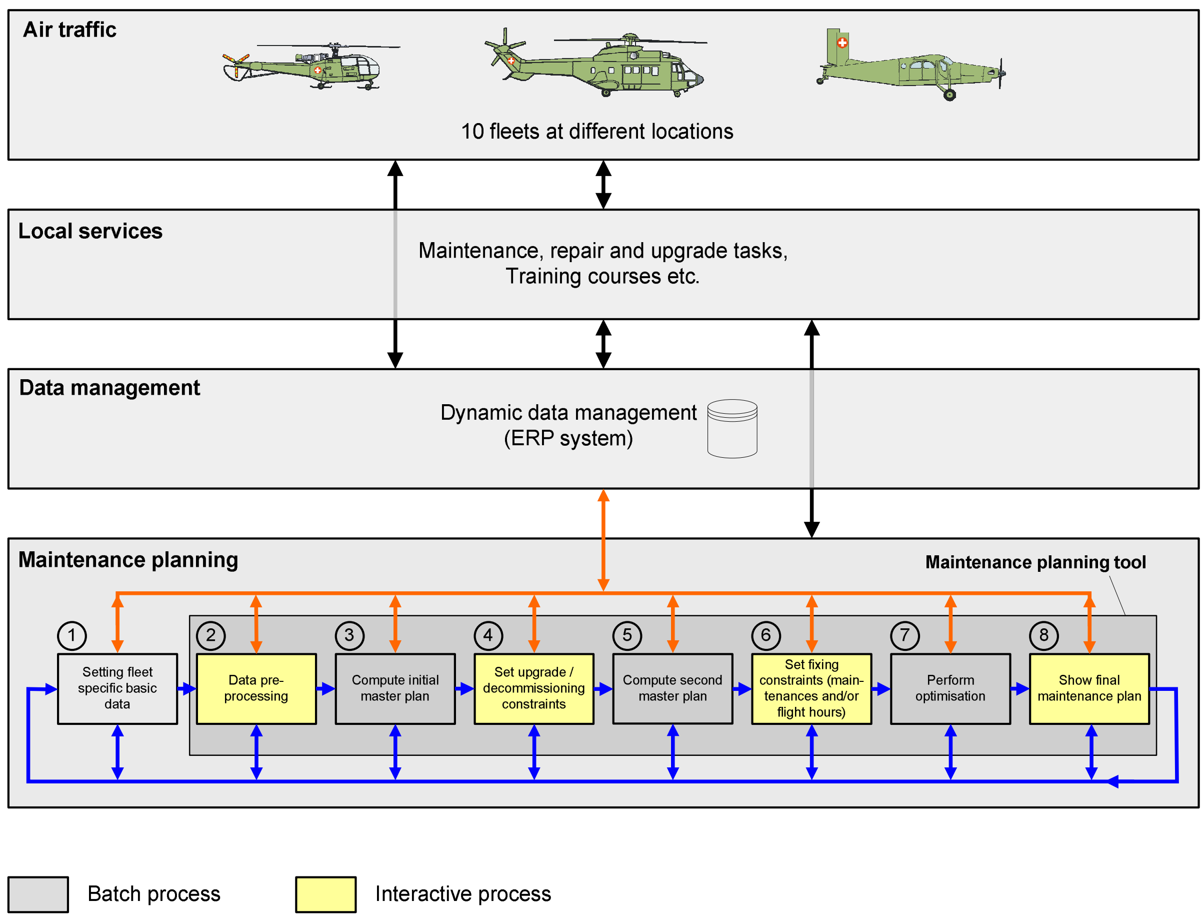
[4] Type of constraint: S → must be strictly met, R → can be relaxed to a certain extent, if required

3. Methodology

3.1 Overview

We identified four operational functions: ′Air traffic′ and ′Local services′ are both decentralised services, whereas ′Data Management′ and ′Maintenance planning′ are centralised tasks. The proposed tool is integrated in the service 'Maintenance planning'. The interrelations between these tasks are illustrated in Figure 1. The focus of this project was on the development of scheduling algorithms, required within the maintenance-planning tool (see bottom of Figure 1).

Figure 1 Overview of the operative air traffic framework. Within the rectangle containing the maintenance planning steps (bottom), blue arrows indicate processes and orange arrows indicate data flows. Black/grey arrows indicate general work-flows.



To clarify the planning procedure, a short description is presented in Table 3. However, only the two crucial steps are explained in detail subsequently: In section 3.2, we introduce the computation of the so-called 'Master plan' (MPL[5]; steps 3 and 5 in Figure 1) and in section 3.3, we describe the optimisation task (OPT[6]; step seven in Figure 1).

Table 3 The eight steps of the maintenance planning procedure.

Step #	Description
1	Set fleet specific basic data (duration of MAs, rules for MA mergings etc.): This step needs to be performed rarely and therefore is not part of the tool.
2	Data pre-processing: This step allows the operator (i) to check and validate the ERP data and (ii) to perform adjustments on these, if required.
3	Compute initial 'Master plan' (MPL): The MPL includes (i) performing the mergings (if required) and (ii) generates a first plan with the CBMAs and UBMAs together with their appropriate tolerance ranges for shifting. (Details see section 3.2.)
4	Set upgrades/decommissioning constraints (optional): In interactive mode, the operator can insert periods with planned upgrades or decommissionings for specific aircrafts.
5	Compute second MPL (only required if step 4 was performed): Since upgrades and/or decommissionings (set in optional step 4) can change the fraction of available aircrafts substantially, it is important, to rerun the MPL task. The algorithm is very similar to the one performed in step 3.
6	Fixing of MAs and/or flying hours (optional): In practice, local services ask the operator (i) to perform some MAs at certain periods and/or (ii) to use one or more aircrafts to some predefined conditions (flying hours, period). To include this in the planning procedure, the operator can shift and/or fix maintenance actions as well as set required flying hours for specific aircrafts and periods.
7	Perform optimisation (OPT): Based on the inputs from previous steps, the optimisation is performed. The main intentions are to position MAs optimally and to distribute flying hours evenly. For a detailed description, see section 3.3.
8	Show final maintenance plan: The MAs as well as the recommended number of flying hours per aircraft per week are visualised in EXCEL spreadsheets (one per year). The required number of flying hours is important for the local services when setting up flight plans.

The process of generating a MPL consists, among other things, of positioning the CBMAs and UBMAs according to current ERP data, a procedure of merging CBMAs and UBMAs to

[5] We will use the abbreviation MPL for 'Master plan' throughout this document

[6] We will use the abbreviation OPT for 'Optimisation' throughout this document

minimise the overall maintenance time and to allocate inter-maintenance flying hours according to the regulations. The optimisation task comprehends finding positions of the MAs such that (i) the various operational constraints are met (according to Table 2), (ii) the resulting maintenance capacity requirements per fleet have a minimum number of exceedings, and (iii) both the maintenance requirements and the flying hours per aircraft per week have a low variability. For the sake of simplicity, we have omitted a fleet index in the rest of this document.

To complete this overview, we list the properties of MAs with some short explanations in Table 4.

Table 4 Properties of maintenance actions.

#	Property	Type[7]	Description
1	Code	U/C	Each maintenance type has a unique code: UBMAs with 1**, CBMAs with 2**. ** is an increasing number reflecting that MAs are sorted in ascending order, according to their duration.
2	Duration	U/C	Duration of the maintenance action
3	Repetition time	U	The number of flying hours after which the same UBMA must be performed again
4	Interval tolerance	U	The tolerance (±) for the repetition time of an UBMA
5	Calendar time	C	The time between two successive CBMAs
6	Weight	U/C	According to the maintenance capacities required, each MA is given a certain weight.
7	Tolerance range	U/C	For UBMAs, the shifting tolerances are the same in positive and negative direction. However, for CBMAs we explicitly distinguish between positive and negative tolerance. Thus, for #7 we have the positive tolerance range for a CBMA and for #8 the negative one.
8	Negative tolerance range	C	see #7

[7] Maintenance types: U → holds for UBMAs only, C → holds for CBMAs only, U/C → holds for both types

3.2 Computation of the 'Master plan'

In Figure 2, the procedure for computing the MPL is shown. No distinction is made between the procedures involved in generating the initial and the second master plan (see Figure 1, steps 3 and 5).

Figure 2 Extended pseudo code describing the computation of the MPL

Import data (calendar, global parameters, fleet specific data, quarterly flying hour budgets, ERP data etc.)

Determine vectors containing codes of applicable UBMAs and CBMAs: $\mathbf{u} = [u_1, \ldots, u_{k,} \ldots, u_K]^{\mathrm{T}}$ and $\mathbf{d} = \mathbf{u}$ if fleet has only UBMAs; $\mathbf{u} = [u_1, \ldots, u_{k,} \ldots, u_K]^{\mathrm{T}}$, $\mathbf{c} = [c_1, \ldots, c_{l,} \ldots, c_L]^{\mathrm{T}}$, and $\mathbf{d} = [\mathbf{u}^{\mathrm{T}}\ \mathbf{c}^{\mathrm{T}}]^{\mathrm{T}}$, if fleet has both UBMAs and CBMAs; $[\cdot]^{\mathrm{T}}$ indicates the transpose of a vector or matrix, respectively).

Compute the nominal the flying hours per week (for the whole fleet) over the whole planning horizon → vector $\mathbf{h}^{\mathrm{nom}}$ (size $[W \times 1]$).

Write (i) running MAs (UBMAs *and* CBMAs) and all CBMAs, (ii) special services, and (iii) pending special services (decommissionings, upgrade services) to maintenance plan

Set $j^{\mathrm{start}} = 1$

While $i < W - 1$ (W : number of weeks within planning horizon)

  For $j = j^{\mathrm{start}}$ to $W - 1$

    Remove all UBMAs entries that start in weeks $> j$, to make ensure that the UBMAs can be correctly written after a successful merging

    For $k = 1$ to N (N : number of aircrafts in fleet)

      If positive tolerance of next CBMA is reached

        Determine the optimal UBMA of aircraft k within an allowed range, i.e. the one, which has the largest duration *and* results in the lowest deviation from the original UBMA position

        If an UBMA is found (i.e., a merging gets possible)

          Perform merging and write CBMA in maintenance plan

          Determine new starting week j^{start}

          Set binary variable v to 1, signalling a successful merging

        Else

          Write code of next CBMA in maintenance plan

        End

      Else

        If repetition time of next UBMA is reached

          Write code of next UBMA in maintenance plan

        End

      End

    End

    Update variables and store all data of week j

    if $v = 1$ (i.e. a merging was performed)

      Exit for-loop (j)

    End

  End

  Set $v = 0$

  If $j = W - 1$

    Stop MPL computation and exit

  End

End

Export final maintenance plan to file server

In the procedure described in Figure 2, the following aspect is of special importance: Once the algorithm has found two MAs for merging, the merger is performed resulting MA is written to the maintenance plan. To consider the new number of available aircrafts, the affected range is determined and the computation starts again at the first week (j^{start}) before this range. This ensures that, especially for small fleets, the mergers do not lead to additional variabilities.

3.3 Performing the optimisation task

The optimisation task consists of finding the optimal positions of the MAs such that (i) the various operational constraints are met (according to Table 2), (ii) the resulting maintenance capacity requirements per fleet have a minimum number of exceedings, and (iii) both the maintenance requirements and the flying hours per aircraft per week have a low variability. Section 3.3.1 gives a general overview of the optimisation task. In section 3.3.2, we discuss the computation of the optimal positions. In section 3.3.3, we have a closer look on balancing the flying hours between adjacent quarters.

3.3.1 General procedure

In this section, we give a general overview of the optimisation procedure. As for the computation of the MPL, we do this with pseudo code as shown in Figure 3.

Figure 3 Extended pseudo code describing the optimisation procedure

Import data (calendar, global parameters, fleet specific data, quarterly flying hour budgets, ERP data etc.)

Determine vectors containing codes of applicable UBMAs and CBMAs: $\mathbf{u} = [u_1, \ldots, u_{k,} \ldots, u_K]^{\mathrm{T}}$ and $\mathbf{d} = \mathbf{u}$ if fleet has only UBMAs; $\mathbf{u} = [u_1, \ldots, u_{k,} \ldots, u_K]^{\mathrm{T}}$, $\mathbf{c} = [c_1, \ldots, c_{l,} \ldots, c_L]^{\mathrm{T}}$, and $\mathbf{d} = [\mathbf{u}^{\mathrm{T}}\ \mathbf{c}^{\mathrm{T}}]^{\mathrm{T}}$, if fleet has both UBMAs and CBMAs; $[\cdot]^{\mathrm{T}}$ indicates the transpose of a vector or matrix, respectively).

Compute the nominal the flying hours per week (for the whole fleet) over the whole planning horizon → vector $\mathbf{h}^{\text{nom}}$ (size $[W \times 1]$).

Read data from 'Master plan (MPL)' and build matrix $\mathbf{F}$ (size $[N \times W]$, N : number of aircrafts in fleet, W : number of weeks within planning horizon), comprehending all MAs together with their appropriate tolerance ranges.

Determine all non-fixed MAs in $\mathbf{F}$ and compute unsorted priority table $\mathbf{P}^{\text{unsort}}$ (size $[M \times 5]$, where M is the total number of non-fixed MAs). Each row contains of the following information for each MA m ($m = 1 \ldots M$):

Column 1: Aircraft number $n^{(m)}$ to which MA m belongs, with $n^{(m)} \in \{1 \ldots N\}$

Column 2: Maintenance type $d^{(m)}$ of MA m, with $d^{(m)} \in \{d_1, \ldots, d_s, \ldots, d_K\}$ (if only UBMAs) or $d^{(m)} \in \{d_1, \ldots, d_s, \ldots, d_K, d_{K+1}, \ldots, d_{K+L}\}$ (if UBMAs and CBMAs)

Column 3: Earliest possible beginning of MA m : $t^{(m)}_{\text{earliest}}$

Column 4: Initial position of the MA m : $t_0^{(m)}$

Column 5: Latest possible beginning of MA m : $t_{\text{latest}}^{(m)}$

Sort matrix $\mathbf{P}^{\text{unsort}}$ according to (i) maintenance type $d^{(m)}$ (in descending order), (ii) $t_{\text{earliest}}^{(m)}$ (in ascending order), and (iii) aircraft number (in ascending order) → matrix $\mathbf{P}^{\text{sort}}$

For all rows in matrix $\mathbf{P}^{\text{sort}}$ (from highest to lowest priority)

Compute optimal position $t_*^{(m)}$ for each MA (details see section 3.3.2 below)

End

Compute nominal flying hours between MAs, considering fixed entries and/or special services and enter again the inter-maintenance flying hours (weighted by $\mathbf{h}^{\text{nom}}$ to get an optimal basis for the subsequent steps)

Compute actual and nominal flying hours per quarter and $\mathbf{f} = [f_1,\ldots,f_q,\ldots f_Q]^{\text{T}}$, containing the flying hour differences (actual minus nominal) per quarter, where $q = 1\ldots Q$ and Q denotes the overall number of quarters within the planning horizon.

Compute the transfer of flying hours between adjacent quarters to meet the quarterly flying hour budgets (details see section 3.3.3)

Compute limiting the flying hours to a (fleet-specific) $h^{\max}$ (details see section 3.3.4)

Compute performance indicators

Export final maintenance plan to file server

The optimization algorithm computes ideal positions for all MAs disposable for shifting. The positions of MAs running at the time of optimization, or those fixed by the user are not changed by the optimization.

3.3.2 Determining optimal positions of maintenance actions

The optimum positions $t_*^{(m)}$ are determined for each maintenance action m according to the priorities assigned:

$$t_*^{(m)} = \arg\min_t \sum_{i=1}^{3} w_i \, p_{i,t}^{(m)} ,$$

where $p_{i,t}^{(m)}$ $(i = 1\ldots3)$ are penalty functions and w_i their respective weights. Function $p_{1,t}^{(m)}$ penalizes the extent to which MA m is shifted from $t_0^{(m)}$, and reflects that any shift away from the original position leads to a more uneven distribution of flying hours over time. Function $p_{2,t}^{(m)}$ penalizes the degree to which capacity requirements for maintenance m overlap with those demanded by the previous $m-1$ maintenance actions. Finally, function $p_{3,t}^{(m)}$ determines conflicts with user-set weekly flying hours. In Figure 4, we show the sequential computation scheme to determine an optimal position of each relocatable UBMA.

Figure 4 Extended pseudo code that describes the computation of the optimal positions of the maintenance actions. The following notations hold: l_m, c_j and e_j are the duration of MA m computed by the MPL, the total capacities required in week j by $m-1$ MAs already set and a binary variable that indicates the presence or absence of user-set weekly flying hours during week j for aircraft $n^{(m)}$ (explicitly defined by MA m), respectively.

Read matrix $\mathbf{P}^{\text{sort}}$
For m = 1 to M
  For j = 1 to W
    Compute capacity requirements for week j up to MA $m-1$. Vector $\mathbf{r}$ contains the maintenance resources required by maintenance type s and is defined as $\mathbf{r}=[r_1,\ldots,r_s,\ldots,r_K]^{\text{T}}$ (only UBMAs) or $\mathbf{r}=[r_1,\ldots,r_s\ldots,r_K,r_{K+1},\ldots r_{K+L}]^{\text{T}}$ (both UBMAs and CBMAs), where $0<r_s\le 1$ $(\forall s)$ holds. K denotes the total number of UBMAs and L is the total number of CBMAs.
      For i = 1 to N

$$c_j = c_j + \begin{cases} r_s & \text{if } \min\limits_s \left| F_{ij}^{(m-1)} - d_s \right| = 0 \\ 0 & \text{otherwise (i.e., } F_{ij}^{(m-1)} \text{contains no MA)} \end{cases},$$

      where $F_{ij}^{(m-1)}$ denotes the maintenance plan entry of aircraft i, in week j after MA $m-1$ and d_s denotes the maintenance code at position s in vector $\mathbf{d}$.
      End
    Compute binary variable that reflects the availability of fixed flying hours in week j for aircraft $n^{(m)}$:

$$e_j = \begin{cases} 1 & \text{if } h^{\text{down}} < F_{n^{(m)},j}^{(m-1)} < h^{\text{up}} \\ 0 & \text{otherwise (i.e., } F_{n^{(m)},j}^{(m-1)} \text{contains no fixed flying hours)} \end{cases},$$

    where h^{down} and h^{up} denote the lower and upper limits of flying hours/aircraft/week ($h^{\text{down}}=0$, $h^{\text{up}}=99$). Please note that $h^{\text{up}}>h^{\max}$ holds.
  End
  For t = $t_{\text{earliest}}^{(m)}$ to $t_{\text{latest}}^{(m)}$
    Compute penalty function $p_{1,t}^{(m)}$, $p_{2,t}^{(m)}$, and $p_{3,t}^{(m)}$, respectively

$$p_{1,t}^{(m)} = \left| t - t_0^{(m)} \right|,\ p_{2,t}^{(m)} = \sum_{j=t}^{t+l_m-1} c_j \text{, and } p_{3,t}^{(m)} = \sum_{j=t}^{t+l_m-1} e_j,$$

    where the weight coefficients w_1, w_2, and w_3 were set to 0.25, 1 and 1000, respectively to reflect that capacity constraints are more critical than unevenly distributed flying hours and that user-set weekly flying hours need to be respected absolutely.
  End
  Compute the optimal position of MA m:

$$t_*^{(m)} = \arg\min_t \left(w_1 p_{1,t}^{(m)} + w_2 p_{2,t}^{(m)} + w_3 p_{3,t}^{(m)} \right)$$

  Write code $d^{(m)}$ of MA m to positions $t_*^{(m)},\ldots,t_*^{(m)}+l_m-1$ for aircraft $n^{(m)}$ in maintenance plan $\mathbf{F}^{(m)}$
End

Set $c_j^{\text{act}} = c_j,\ j=1,\ldots,W$

3.3.3 Balancing quarterly hours between adjacent quarters

Due to the previous actions, the sum of quarterly flying hours of the whole fleet usually deviate from their nominal values. This shall be corrected with this step.

Figure 5 Extended pseudo code of the balancing procedure for transfer flying hours between adjacent quarters. Weeks are numbered serially, starting with week one at 01.01.2003.

Read vector $\mathbf{f}^{\text{init}}$, containing the flying hour differences per quarter and set $\mathbf{f} = \mathbf{f}^{\text{init}}$

For $q = 1$ to $Q-1$ (Q: number of quarters within planning horizon)

$\mathbf{w}_q = [w_q^{\min}, \ldots, w_q^{\max}]^{\mathrm{T}}$, $\mathbf{w}_{q+1} = [w_{q+1}^{\min}, \ldots, w_{q+1}^{\max}]^{\mathrm{T}}$: week numbers of quarters q and $q+1$ respectively

For $i = 1$ to N

Determine week numbers in quarters q and $q+1$, where MAs are performed for aircraft i:

$\mathbf{m}_{iq} = [m_{i1}, \ldots, m_{ij_1}, \ldots, m_{in_1}]^{\mathrm{T}}$, $\mathbf{m}_{i,q+1} = [m_{i1}, \ldots, m_{ij_2}, \ldots, m_{in_2}]^{\mathrm{T}}$ and

Determine week number, where the last MA ends (m_{iq}^-) and/or the next MA begins ($m_{i,q+1}^+$):

$$m_{iq}^- = \begin{cases} \max\limits_{m_{ij_1} \le \max(\mathbf{w}_q)} m_{ij_1} & \text{if } \mathbf{m}_{iq} \ne [\,] \\ \min(\mathbf{w}_q) - 1 & \text{otherwise} \end{cases} \quad \text{and} \quad m_{i,q+1}^+ = \begin{cases} \min\limits_{m_{ij_2} \ge \min(\mathbf{w}_{q+1})} m_{ij_2} & \text{if } \mathbf{m}_{i,q+1} \ne [\,] \\ \max(\mathbf{w}_{q+1}) + 1 & \text{otherwise} \end{cases}.$$

Update sums of flying hours performed after the last MA ($S_q^{(r)}$) in quarter q and before the next MA ($S_{q+1}^{(l)}$) in quarter $q+1$, i.e., add the contributions from aircraft i:

$$S_q^{(r)} = S_q^{(r)} + \begin{cases} \sum\limits_{k=m_{iq}^-+1}^{\max(\mathbf{w}_q)} F_{ik}^{(m)} & \text{if } m_{iq}^- < \max(\mathbf{w}_q) \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$S_{q+1}^{(l)} = S_{q+1}^{(l)} + \begin{cases} \sum\limits_{k=1}^{m_{i,q+1}^+-1} F_{ik}^{(m)} & \text{if } m_{i,q+1}^+ > \min(\mathbf{w}_{q+1}) \\ 0 & \text{otherwise} \end{cases}.$$

End

Compute coefficients, which determine the proportion of available flying hours per quarter that needs to be transferred between quarters q and $q+1$:

$$\phi_q = \left(S_q^{(r)} - f_q\right) \big/ S_q^{(r)} \quad \text{and} \quad \phi_{q+1} = \left(S_{q+1}^{(l)} + f_q\right) \big/ S_{q+1}^{(l)}$$

If $\phi_q > 0 \wedge \phi_{q+1} > 0$

Perform transfers

For $i = 1$ to N

If $f_q > 0$ (transfer from quarter q to $q+1$ required, $0 < \phi_q < 1$)

$F_{ik_1}^{(m)} = F_{ik_1}^{(m)} \phi_q$, for all $k_1 = m_{iq}^- + 1, \ldots, \max(\mathbf{w}_q)$

$\Delta_1 = \left(1 - \phi_q\right) \sum_{k_1} F_{ik_1}^{(m)}$

$F_{ik_2}^{(m)} = F_{ik_2}^{(m)} + \Delta_1 \big/ [m_{i,q+1}^+ - \min(\mathbf{w}_{q+1})]$, for all $k_2 = \min(\mathbf{w}_{q+1}), \ldots, m_{i,q+1}^+ - 1$

Else (transfer from quarter $q+1$ to q required, $0<\phi_{q+1}<1$)

$F_{ik_2}^{(m)} = F_{ik_2}^{(m)}\phi_{q+1}$, for all $k_2 = \min(\mathbf{w}_{q+1}),\ldots,m_{i,q+1}^{+}-1$

$\Delta_2 = \left(1-\phi_{q+1}\right)\sum_{k_2} F_{ik_2}^{(m)}$

$F_{ik_1}^{(m)} = F_{ik_1}^{(m)} + \Delta_2 \big/ [\max(\mathbf{w}_q) - m_{iq}^{-}]$, for all $k_1 = m_{iq}^{-}+1,\ldots,\max(\mathbf{w}_q)$

End

End

End

Update vector **f**

$S_q = S_q^{(l)} + S_q^{(r)} - f_q$ and $S_{q+1} = S_{q+1}^{(l)} + S_{q+1}^{(r)} + f_q$

$f_q = S_q - S_q^{\text{nom}},\ \forall q$

End

3.3.4 Introducing an upper limit of flying hours per aircraft per week

After balancing the flying hours between quarters, for some aircrafts the number of flying hours are above a certain threshold. To prevent changes in flying hours between successive weeks (e.g., difference of more than four hours) we introduced the following procedure: For weeks with exceedings, the amount above the threshold is allocated to the adjacent weeks. The entries are weighted with the nominal flying hours for the whole fleet in the appropriate weeks to ensure that the previously balanced quarterly flying hour budgets are not changed too much again. The smaller the threshold is set, the more difficult it gets to distribute the remaining flying hours to the adjacent weeks. In other words: deviations between the actual and the nominal flying hours per quarter increase again with decreasing upper limits. The influence on the deviation depends strongly on the fleet and on number and type of constraints set. In section 4, we show the influence of the threshold on the deviations for two fleets with some typical configurations. The inter-maintenance flying hours are not changed due to this procedure.

4. Results

In this section, we present the achievements of this project. In section 4.1, we introduce the performance criteria applied. In section 4.2, we briefly describe the fleets investigated and in sections 4.3 and 4.4, we show the results after computing the MPL and performing the OPT, respectively. To conclude, in section 4.5 we present in brief the implemented tool (GUI).

4.1 Performance criteria

To get an overview, in Table 5 we link the parameters and the performance criteria, respectively, to the steps performed. It is important to note, that finding the optimal position of the MAs and balancing the flying hours between adjacent quarters are independent steps. This holds also for parameters $h^{\max}$ (maximum flying hours per aircraft per week) and $c_t^{\lim}$ (maximum available maintenance capacities), as shown in Table 5.

Table 5 Parameters and performance criteria. All plans satisfied the strict constraints. Therefore, only the weak constraints are considered here.

Step	Parameter[8]			Performance criterion	
	$h^{\max}$	$c_t^{\lim}$ [9]	UBMA tol.	Criterion	Description
Compute MPL (section 3.2)	0	0	+	f^{merg}	Fraction of achieved to possible number of mergers
Find optimal position of MAs (section 3.3.2)	0	+ +	†	$\Delta^{\text{cap}}\left(c_t^{\lim}\right)$	Exceeding of available maintenance capacities
Perform balancing (section 3.3.3) and limitation (section 3.3.4) of flying hours (both tasks within OPT)	+	0	†	$F(h)$	Cumulative distribution of flying hours for the whole fleet over planning horizon
	+ +	0	†	$\Delta^{\text{fq}}\left(h^{\max}\right)$	Deviation of flying hours per quarter from nominal value

[8] Influence: ++ → strong, + → minor, 0 → none, † → not investigated in detail

[9] $c_t^{\lim}$ does currently not depend on time, so $c_t^{\lim} = C^{\lim}, \forall t$ holds; $C^{\lim} > 0$ denotes a constant (real).

In the following, we briefly introduce the four performance criteria according to Table 5.

Mergers completed

The fraction of achieved to possible number of mergers f^{merg} is computed as follows:

$$f^{\text{merg}} = \frac{1}{B}\sum_{b=1}^{B}\gamma_b ,$$

where B is the number of CBMAs available for merging, for the whole fleet within the planning horizon, and γ_b is a binary variable, computed as

$$\gamma_b = \begin{cases} 1 & \text{if CBMA } b \text{ was merged successfully} \\ 0 & \text{otherwise} \end{cases} .$$

Deviations of actual from nominal quarterly flying hours

The deviation between the actual and nominal sum of quarterly flying hours for the planning horizon is computed as follows:

$$\Delta^{\text{fq}}\left(h^{\max}\right) = 100\frac{\sum_{q=1}^{Q}\left| S_q^{\text{act}}\left(h^{\max}\right) - S_q^{\text{nom}}\right|}{\sum_{q=1}^{Q} S_q^{\text{nom}}},$$

where $S_q^{\text{act}}\left(h^{\max}\right)$ denotes the actual flying hours, depending on the upper flying hour limit $h^{\max}$, S_q^{nom} is the nominal number of flying hours in quarter q, and finally $\Delta^{\text{fq}}\left(h^{\max}\right)$ denotes the overall deviation in percent.

Distribution of the flying hours

As mentioned in section 3.3.4, the flying hours are to have a low variation over time. A good measure to quantify this is the cumulative distribution function (cdf) of the flying hours. For this, the flying hours of the whole fleet in the planning horizon were considered:

$$F(h) = \sum_{h_i \le h} p(h_i) ,$$

where $F(h)$ is the cumulative distribution function, h_i denotes the computed flying hours, with $i = 1,\ldots,R$ (R: the number of bins of h), $p(h_i)$ denotes the relative frequency of h_i ($0 \le p(h_i) \le 1$, $\sum_{h_i} p(h_i) = 1$), and h represents the flying hours, for which we compute the cdf.

The aim is to have distributions with low variance, i.e., we require cdfs $F(h)$ without long tails to the right. Thus, we examined $F(h) = 0.995\ldots1$ in detail, i.e., the flying hours lying in the upmost 0.5% (see section 4.4.2).

Number of exceedings of available maintenance capacities

To quantify the number of exceedings as well as their length, the following formula is applied:

$$\Delta^{\text{cap}}\left(c_t^{\text{lim}}\right) = \sum_{e=1}^{n^{\text{ex}}} \left(t_e^{\text{end}} - t_e^{\text{start}} + 1\right)^{\delta} \sum_{t=t_e^{\text{start}}}^{t_e^{\text{end}}} \left(c_t^{\text{act}} - c_t^{\text{lim}}\right),$$

where n^{ex} denotes the number of independent sequences of exceedings, e denotes a specific exceeding, t_e^{start} and t_e^{end} are the begin and end of an exceeding, respectively, and c_t^{act} and c_t^{lim} indicate the actual and the nominal capacity at time t. Finally, δ denotes a constant that determines the weight of penalty function $\left(t_e^{\text{end}} - t_e^{\text{start}} + 1\right)^{\delta}$. In other words, if $\delta > 1$, long exceedings are penalised more than shorter ones; we set $\delta = 2$.

4.2 Test fleets and setup

For the tests presented here, two fleets were investigated in detail. Fleet one consists of UBMAs as well as CBMAs, whereas fleet two has only UBMAs. For specific demonstration purposes, a third fleet is used (see section 4.4.2, Table 7).

Table 6 Nominal flying hours for fleet 1 and 2, respectively.

year y →	2006		2007		2008		2009		2010	
fleet # →	1	2	1	2	1	2	1	2	1	2
quarter q ↓	[h]	[h]	[h]	[h]	[h]	[h]	[h]	[h]	[h]	[h]
1	690	550	951	825	875	1000	675	860	675	980
2	1254	750	1245	455	875	840	675	775	675	680
3	1113	700	1023	825	875	760	675	825	675	810
4	849	850	780	650	875	850	675	550	675	740
sum/year →	3906	2850	3999	2755	3500	3450	2700	3010	2700	3210

For both fleets, the tests were carried out with typical configurations. The number of manual constraints was set low to make the results comparable.

Higher deviations can arise, depending on the number of additional constraints. Especially for small fleets with a large number of fixed maintenance actions, special services or fixed flying hours, larger deviations in the quarterly flying hours are expected. Nevertheless, for all tests performed, the deviations were always within meaningful ranges.

We will not explain the constraints as well as the ERP data in detail here.

4.3 Some comments on the 'Master plan'

Computing the MPL includes (i) the entry of the MAs and (ii) performing mergings between feasible CBMAs and UBMAs (for those fleets with both types of MAs).

The quality of a master plan is measured mainly by the fraction of achieved to possible mergers (f^{merg}). For the fleets considered, $f^{\text{merg}} > 0.95$, i.e., nearly all possible mergers could be conducted successfully. However, in a few cases this was not possible because the ranges for shifting the CBMAs were too small. Since this is a strict constraint defined by the regulations, the number of mergers cannot be increased without changing this constraint.

To summarise we can say, that the performance of this task is very good with respect to MA entries and mergings. Although computing the MPL is important, the performance of the approach is measured after the optimisation task, which is done in the following section.

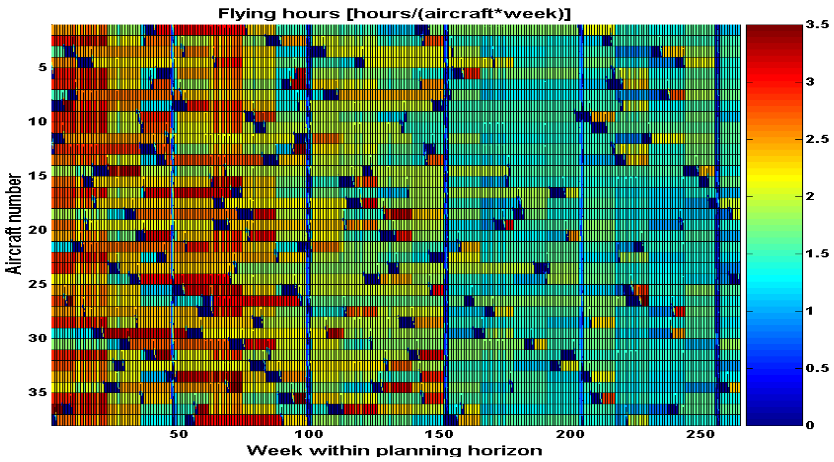
4.4 Assessment of maintenance plan after optimisation

In this section, we assess the quality of the maintenance plans after the optimisation according to the performance criteria defined in section 4.1. It is important to state at this point that we only get meaningful results, since *all* tasks (MPL, balancing quarterly flying hours, and limitation of the flying hours per aircraft per week) can be carried out successfully.

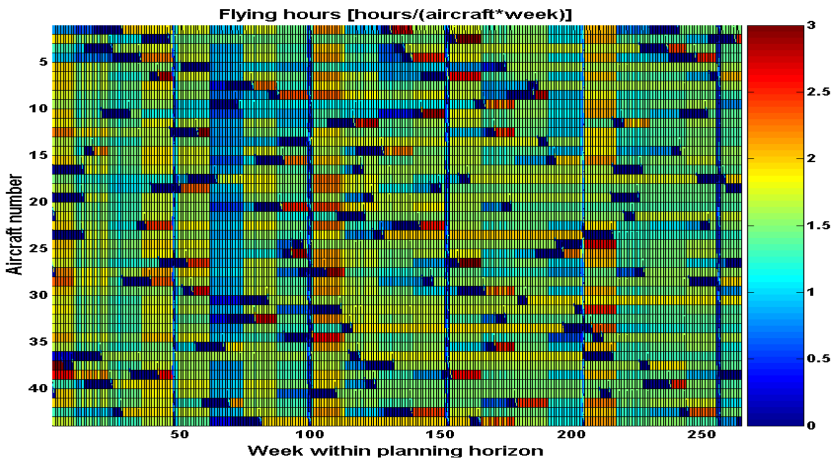
4.4.1 Flying hours

To begin, we have a look at the resulting flying hours for the two fleets and the whole planning horizon under conditions as defined in section 4.2. As shown in Figure 6a, fleet one has a high demand in the first 7 quarters, which is reflected by the relatively large number of flying hours in this range. In contrary, fleet 2 (see Figure 6b) has significantly less flying hours in the sixth quarter.

Figure 6 Flying hours for the configurations according to section 4.2: (a) Fleet 1, with $h^{\max}$ = 3.5 hours, (b) Fleet 2, with $h^{\max}$ = 3 hours. Cells with zero flying hours indicate weeks with either MAs or holidays only (e.g., Christmas/New Year).



(a)



(b)

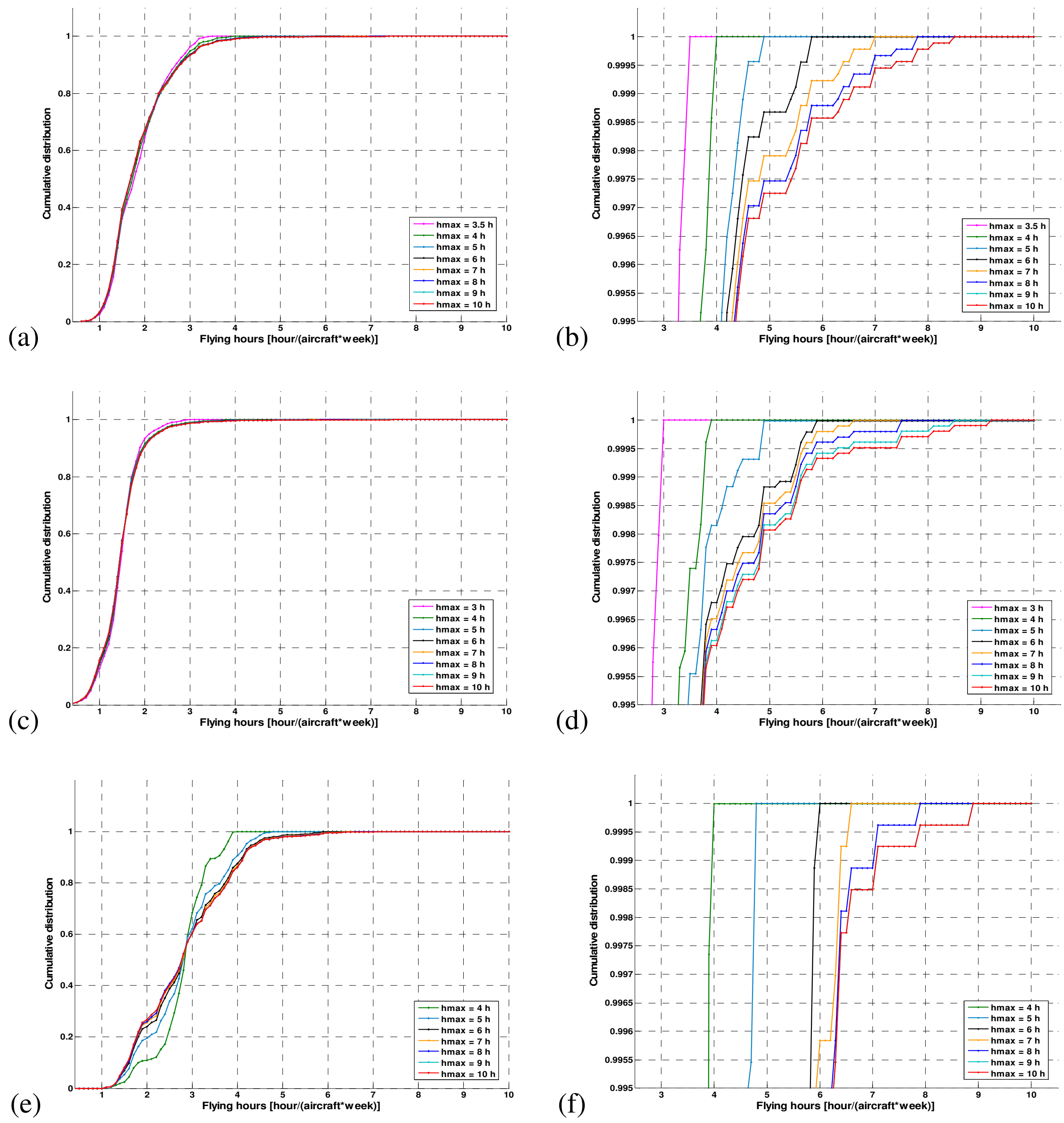
4.4.2 Distribution of flying hours per aircraft per week

In Figure 7, the cumulative distribution functions of flying hours subject to $h^{\max}$ are shown for three fleets. In general, the intention is to have a small number of entries that deviate largely from the mean.

From Figure 7 (b, d, and f) we can determine the range of flying hours that comprise the upmost 0.5% of all cases. For $h^{\max}$ = 10 hours, we get the following ranges: (i) fleet 1: 4.3 to 8.4 hours, (ii) fleet 2: 3.7 to 9.1 hours, and (iii) fleet 3: 6.2 to 8.8 hours. The appropriate mean is 1.77, 1.39, and 2.62 hours, respectively. We see that the number of flying hours lying in the upmost 0.5% of all cases is low, which is well acceptable from an operational point of view.

Especially for fleets with a small number of aircrafts (fleet 1: 37, fleet 2: 43, and fleet 3: 11 aircrafts) the deviations are somewhat larger since the influence of each MA compared to the number of available aircrafts is higher for small fleets. This fact is well illustrated by comparing Figure 7e to Figure 7a and 7c, respectively.

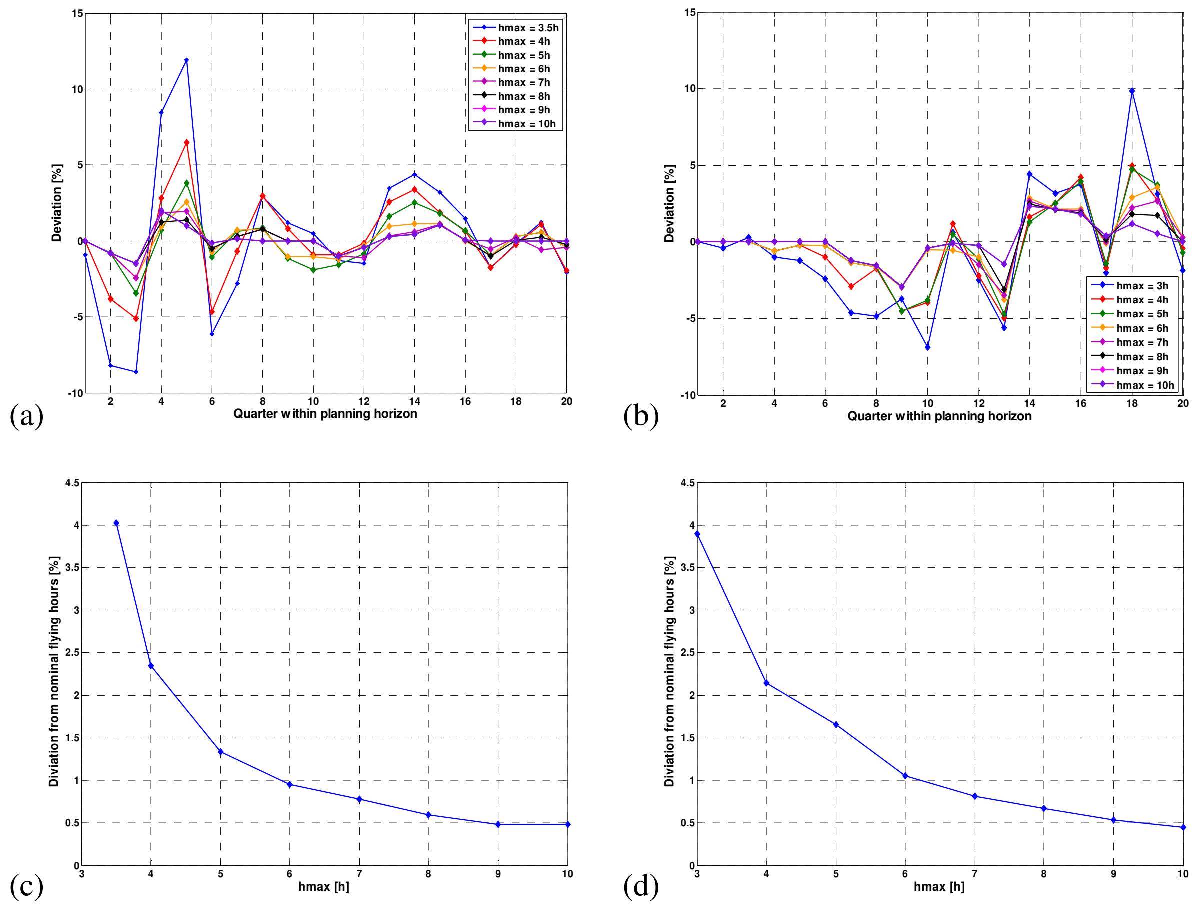
Figure 7 Cumulative distributions of flying hours per aircraft for three fleets: (a, b) top (fleet 1, UBMAs *and* CBMAs), with $h^{\max} = 3.5, 4, 5, \ldots, 10$ hours, (c, d) middle (fleet 2, UBMAs only), with $h^{\max} = 3, 4, \ldots, 10$ hours and (e, f) bottom (fleet 3, UBMAs *and* CBMAs), with $h^{\max} = 4, 5, \ldots, 10$ hours. The left side shows the full cdf, whereas the right side shows only the range $F(h) = 0.995 \ldots 1$.



4.4.3 Deviations from nominal flying hours per quarter

In Figure 8, for fleets 1 and 2 the deviations of the actual from the nominal flying hours per quarter are presented. For both fleets, the deviations become smaller with increasing $h^{\max}$ (see a, b). For $h^{\max} > 6\,\text{hours}$, the deviations are acceptable. The specific characteristics per fleet are mainly determined by the actual configuration. From (c, d) we see, that with increasing $h^{\max}$ the deviations get smaller for both fleets.

Figure 8 Deviations of actual from nominal quarterly flying hours subject to $h^{\max}$. In (a) and (b) the deviations in percent from the nominal values per quarter are shown for fleet 1 and fleet 2, respectively. (c) and (d) show the deviation $\Delta^{\text{fq}}\left(h^{\max}\right)$ of flying hours over the whole planning horizon for fleet 1 and fleet 2, respectively.



From sections 4.4.2 and 4.4.3, we see that a meaningful selection of $h^{\max}$ is a compromise between two opposite requirements: The goal of having a small variation in flying hours requires to chose $h^{\max}$ small. From the viewpoint of small deviations of quarterly flying hours, we need to set $h^{\max}$ above a certain threshold.

Independent of the weight of the two requirements, we recommend to chose $h^{\max}$ in the following range:

$$2.5\frac{\sum_q s_q^{\text{nom}}}{w^{\text{ph}}N} \leq h^{\max} \leq 4\frac{\sum_q s_q^{\text{nom}}}{w^{\text{ph}}N},$$

where s_q^{nom} is the nominal number of flying hours in quarter q, w^{ph} denotes the number of weeks with the planning horizon, i.e., $w^{\text{ph}} \cong 260$, and N is the number of aircrafts of the fleet investigated. The lower boundary reflects the fact that is determined by the deviations

In practice, $\left(1/w^{\text{ph}}N\right)\sum_q s_q^{\text{nom}}$ is slightly lower than the actual mean of the flying hours per aircraft per week, since it does not take into account the MAs. However, to get a rough estimate of the mean, it works well.

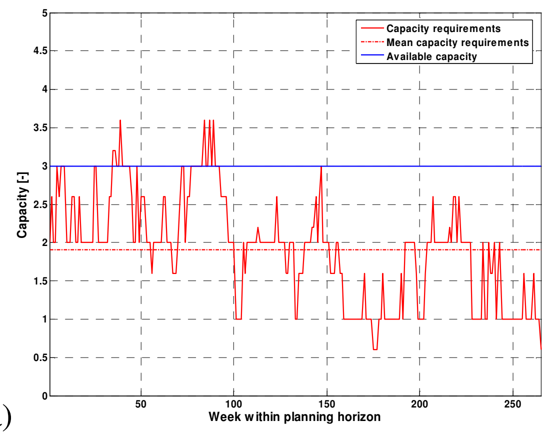
Depending on the individual and/or fleet-dependent weight $h^{\max}$ can be chosen closer to the lower or the upper limit.

In practice, the number, type and range of the constraints need to be considered too when setting this parameter.

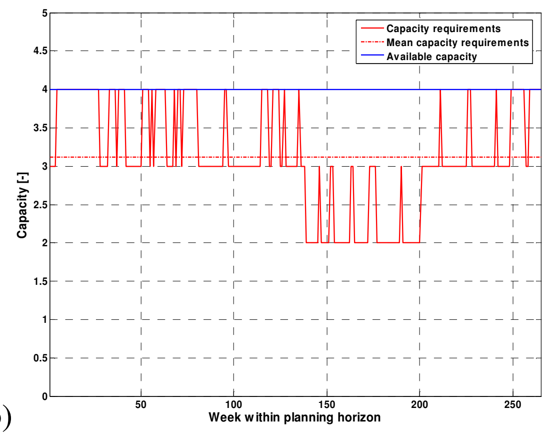
4.4.4 Capacity requirements

The actual capacity requirements and the available capacities are shown in Figure 9. For both fleets, the requirements were met well, given their appropriate configurations.

Figure 9 Capacity requirements after optimisation and available capacity for fleet 1 (a) and fleet 2 (b).



(a)



(b)

For the capacity requirements as shown in Figure 9, the values of $\Delta^{\text{cap}}\left(c_t^{\text{lim}}\right)$ are:

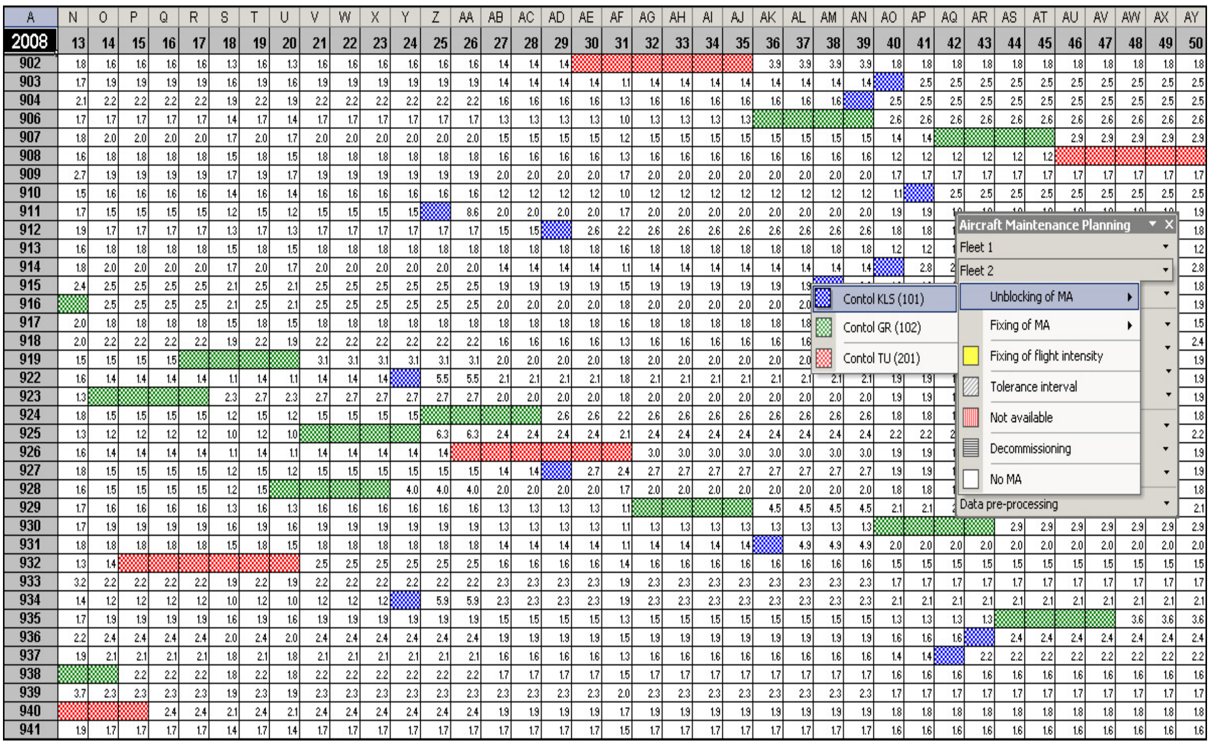
- Fleet 1: $\Delta^{\text{cap}}\left(c_t^{\text{lim}} = C^{\text{lim}} = 3 \text{ (const)}\right) = 4$
- Fleet 2: $\Delta^{\text{cap}}\left(c_t^{\text{lim}} = C^{\text{lim}} = 4 \text{ (const)}\right) = 0$

Both cases are well acceptable, although for fleet one some short exceedings occur. For fleet one, the maintenance capacity provided could be reduced according to $c_t^{\text{lim}} = 2,\ t \geq 100$.

4.4.5 Final maintenance (and flight) plan

Figure 10 shows the final maintenance plan as EXCEL® spreadsheet. For each year in the planning horizon, a separate spreadsheet is provided. Additionally, a customer-specific pull-down menu was implemented to ease and support the interaction processes (see Figure 10, right).

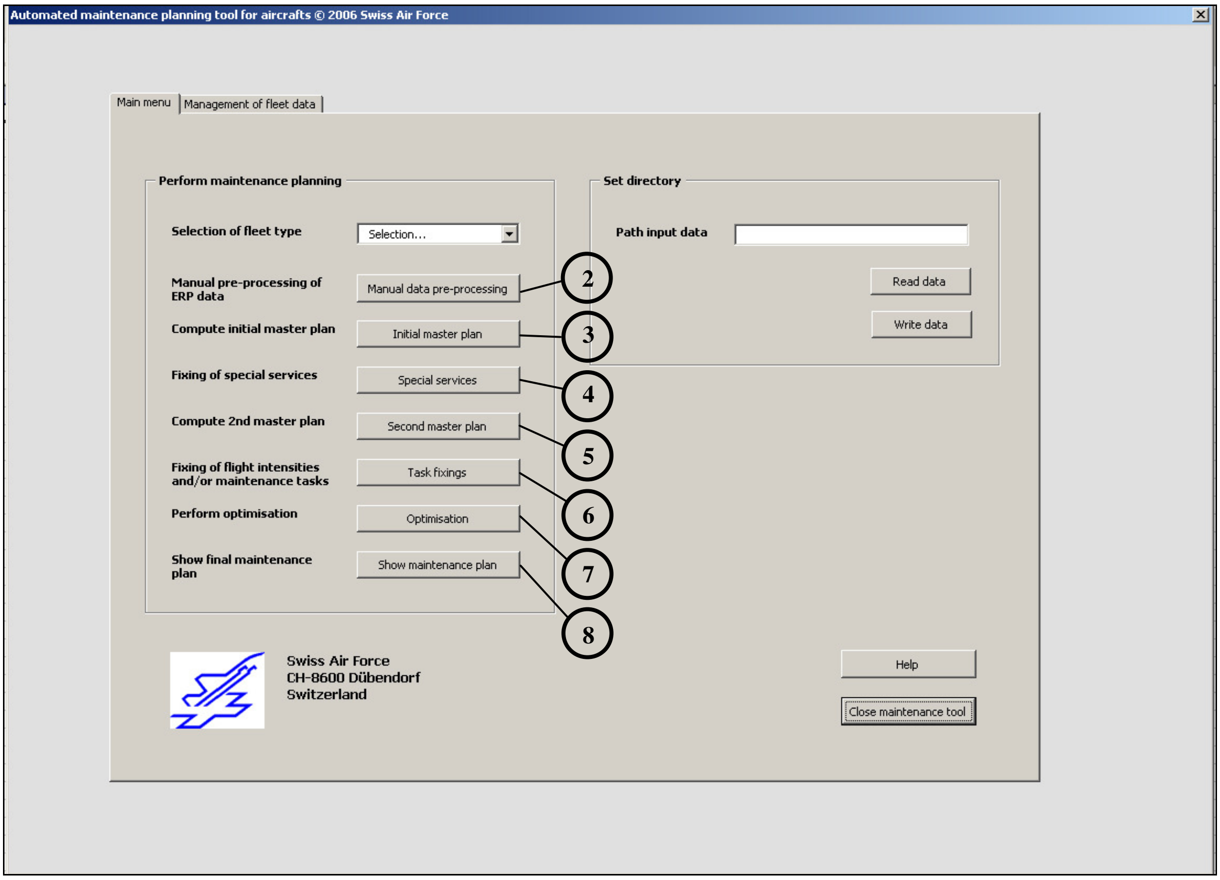
Figure 10 Excerpt of final maintenance/flight plan after optimisation: The cell entries show the maintenance action (each colour represents a specific type of maintenance action) and/or the weekly flying hours, respectively. The first row denotes the planning year and the week numbers, respectively, whereas the first column denotes the aircraft number. In the right half of the figure, the fleet-specific pull-down menu is shown.



4.5 Maintenance Planning Tool

To be compatible with the customers' operating system (MS Windows®) and their familiar office software environment (MS Office), we implemented the software tool in Visual Basic®[10] (VB for MS EXCEL®). The core functionalities (MPL and OPT algorithms) were implemented in MATLAB®[11]. According to the procedure shown in Figure 1, steps 2, 4, 6, and 8 are interactive, whereas steps 3, 5, and 7, run the appropriate MATLAB® executables (see also Figure 11).

Figure 11 Graphical User Interface (GUI) of the maintenance-planning tool: The main menu is in accordance with the procedure presented in Figure 1. Please note that the tool consists of steps 2 to 8 only since constants (step 1) are not changed during standard operation and therefore do not require access via GUI. The second worksheet 'Management of fleet data' allows setting the fleet specific data like quarterly flying hour budgets etc. but is not shown in detail.



[10] Visual Basic and EXCEL are registered trademarks of Microsoft Corporation

[11] MATLAB is a registered trademark of The MathWorks, Inc.

5. Conclusions and Outlook

5.1 Conclusions

We presented a new algorithm for aircraft maintenance scheduling for unscheduled flights. For two fleets we have shown some results and findings. The implemented tool together with its algorithms has shown to work reliable, fast, and with good optimisation results even under heavy constraints, e.g. with various manual settings.

As for most software tools, besides the tasks described in the preceding sections, a substantial part of the project was required (i) to define workflows that meet the requirements of the customer while still being feasible to implement and (ii) to investigate/define the numerous combinatorial special cases and restrictions, mainly due to the large number of manual interaction possibilities.

The tool is in operation at the Swiss Air Force planning central since June 2005. The payback time of this project is about two years. The achieved customer benefits are listed in Table 7.

Table 7 Customer benefits

- Operational tool that fully meets all intentions defined in section 1 (Table 1)
- The time to compute a new maintenance/flight plan, is now within 5 to 15 minutes (depending on the fleet size and the number of constraints), compared to the previously 1.5 to 2 days. This allows for:
    - Fast reaction on events like thunderstorms (Bernese Oberland, August 2005), tsunamis (Sumatra mission, January/February 2005) or forest fires (Leuk, summer 2003) while still meeting all maintenance requirements.
    - Efficient investigation of 'What-If-Analyses' to compare different maintenance strategies
- Automated compliance of regulations like inter-maintenance flying hours etc
- Besides the computation of optimal maintenance plans, the recommended number of flying hours is an important benefit for the local services, since it allows them a much better setup of their flight plans., i.e., the allocation of aircrafts to required flights. It has been shown, that this leads to another substantial cost reduction.

5.2 Outlook

Based on our experiences, we currently see the following main topics for further research:

- Extension of the optimisation procedure such that:
  - The optimal upper limit (h_*^{max}) of the flying hours per aircraft per fleet is determined against both the deviation of quarterly flying hours and the variance of the flying hours (per aircraft per week)
  - The optimisation can be carried out for all fleets to share common maintenance capacities
  - The capacity limit is considered as a strict constraint. However, this might require some changes in the regulations, since the tolerance ranges for shifting the CBMAs are currently very small and thus restrict the flexibility considerably
  - The available maintenance capacity becomes time dependent
- Another interesting aspect is the extension of the procedure to applications other than aircraft maintenance. With the development of a concept for the maintenance of railway fleets, a first step in this direction was made recently by the IDP.

6. Acknowledgements

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